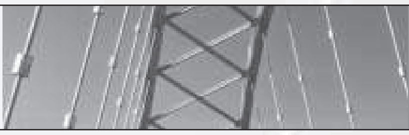
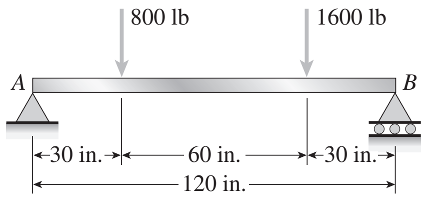


4

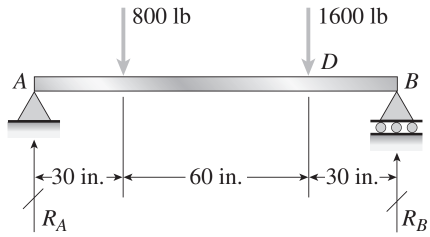
Shear Forces and Bending Moments

Shear Forces and Bending Moments

Problem 4.3-1 Calculate the shear force V and bending moment M at a cross section just to the left of the 1600-lb load acting on the simple beam AB shown in the figure.

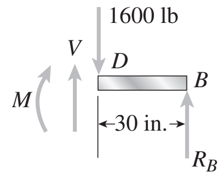


Solution 4.3-1 **Simple beam**



$\Sigma M_A = 0$: $R_B = 1400$ lb
$\Sigma M_B = 0$: $R_A = 1000$ lb

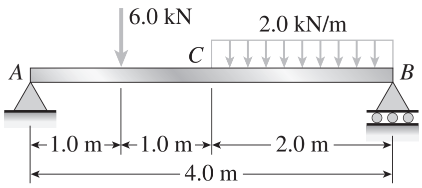
FREE-BODY DIAGRAM OF SEGMENT DB



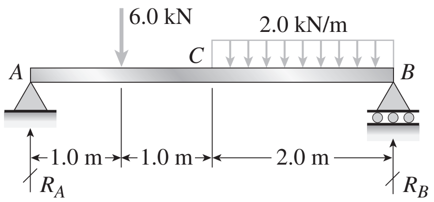
$\Sigma F_{VERT} = 0$: $V = 1600 \text{ lb} - 1400 \text{ lb}$
$= 200$ lb ←

$\Sigma M_D = 0$: $M = (1400 \text{ lb})(30 \text{ in.})$
$= 42{,}000$ lb-in. ←

Problem 4.3-2 Determine the shear force V and bending moment M at the midpoint C of the simple beam AB shown in the figure.

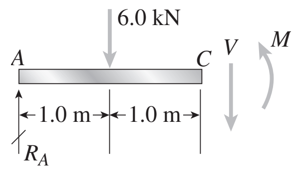


Solution 4.3-2 **Simple beam**



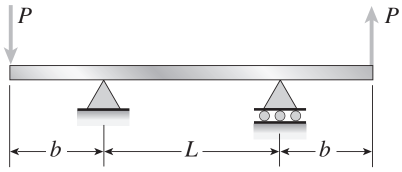
$\Sigma M_A = 0$: $R_B = 4.5$ kN
$\Sigma M_B = 0$: $R_A = 5.5$ kN

FREE-BODY DIAGRAM OF SEGMENT AC

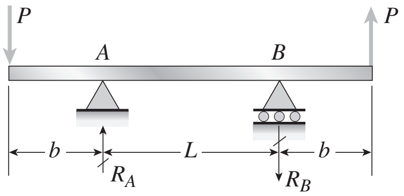


$\Sigma F_{VERT} = 0$: $V = -0.5$ kN ←
$\Sigma M_C = 0$: $M = 5.0$ kN · m ←

Problem 4.3-3 Determine the shear force V and bending moment M at the midpoint of the beam with overhangs (see figure). Note that one load acts downward and the other upward.



Solution 4.3-3 Beam with overhangs

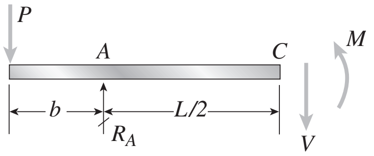


$$\Sigma M_B = 0$$

$$R_A = \frac{1}{L}[P(L + b + b)]$$

$$= P\left(1 + \frac{2b}{L}\right) \quad \text{(upward)}$$

$$\Sigma M_A = 0: \quad R_B = P\left(1 + \frac{2b}{L}\right) \quad \text{(downward)}$$



FREE-BODY DIAGRAM (C IS THE MIDPOINT)

$$\Sigma F_{\text{VERT}} = 0:$$

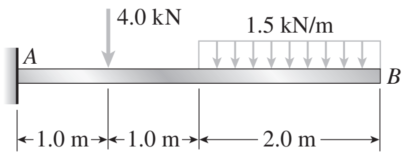
$$V = R_A - P = P\left(1 + \frac{2b}{L}\right) - P = \frac{2bP}{L} \quad \leftarrow$$

$$\Sigma M_C = 0:$$

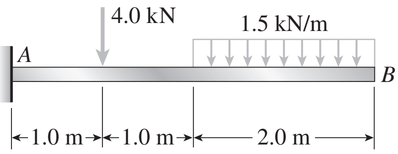
$$M = P\left(1 + \frac{2b}{L}\right)\left(\frac{L}{2}\right) - P\left(b + \frac{L}{2}\right)$$

$$M = \frac{PL}{2} + Pb - Pb - \frac{PL}{2} = 0 \quad \leftarrow$$

Problem 4.3-4 Calculate the shear force V and bending moment M at a cross section located 0.5 m from the fixed support of the cantilever beam AB shown in the figure.

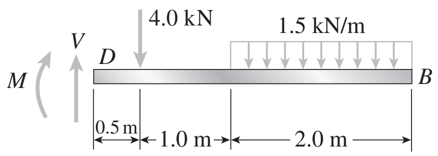


Solution 4.3-4 Cantilever beam



FREE-BODY DIAGRAM OF SEGMENT DB

Point D is 0.5 m from support A.



$$\Sigma F_{\text{VERT}} = 0:$$

$$V = 4.0 \text{ kN} + (1.5 \text{ kN/m})(2.0 \text{ m})$$

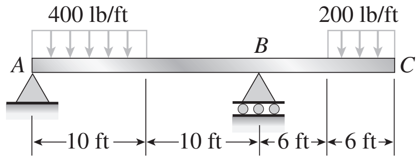
$$= 4.0 \text{ kN} + 3.0 \text{ kN} = 7.0 \text{ kN} \quad \leftarrow$$

$$\Sigma M_D = 0: \quad M = -(4.0 \text{ kN})(0.5 \text{ m}) - (1.5 \text{ kN/m})(2.0 \text{ m})(2.5 \text{ m})$$

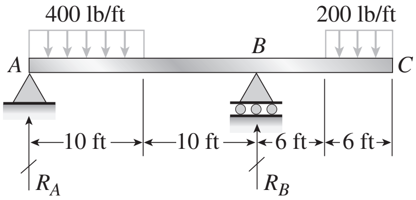
$$= -2.0 \text{ kN} \cdot \text{m} - 7.5 \text{ kN} \cdot \text{m}$$

$$= -9.5 \text{ kN} \cdot \text{m} \quad \leftarrow$$

Problem 4.3-5 Determine the shear force V and bending moment M at a cross section located 16 ft from the left-hand end A of the beam with an overhang shown in the figure.



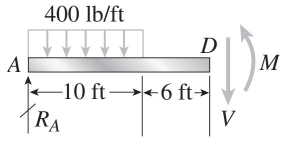
Solution 4.3-5 **Beam with an overhang**



$\Sigma M_B = 0$: $R_A = 2460$ lb

$\Sigma M_A = 0$: $R_B = 2740$ lb

FREE-BODY DIAGRAM OF SEGMENT AD



Point D is 16 ft from support A.

$\Sigma F_{VERT} = 0$:

$V = 2460 \text{ lb} - (400 \text{ lb/ft})(10 \text{ ft})$

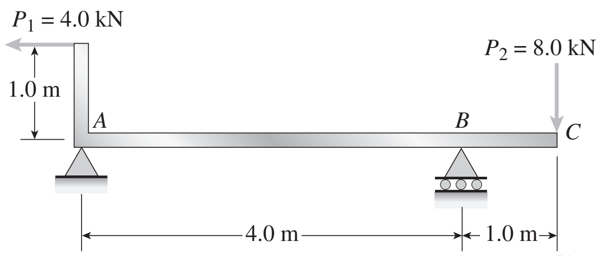
$= -1540 \text{ lb}$ ←

$\Sigma M_D = 0$: $M = (2460 \text{ lb})(16 \text{ ft}) - (400 \text{ lb/ft})(10 \text{ ft})(11 \text{ ft})$

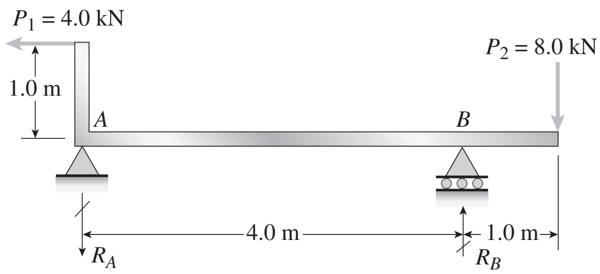
$= -4640 \text{ lb-ft}$ ←

Problem 4.3-6 The beam ABC shown in the figure is simply supported at A and B and has an overhang from B to C. The loads consist of a horizontal force $P_1 = 4.0$ kN acting at the end of a vertical arm and a vertical force $P_2 = 8.0$ kN acting at the end of the overhang.

Determine the shear force V and bending moment M at a cross section located 3.0 m from the left-hand support. (*Note:* Disregard the widths of the beam and vertical arm and use centerline dimensions when making calculations.)



Solution 4.3-6 **Beam with vertical arm**

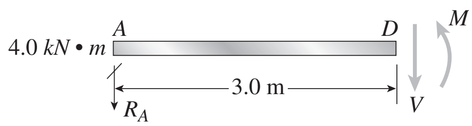


$\Sigma M_B = 0$: $R_A = 1.0$ kN (downward)

$\Sigma M_A = 0$: $R_B = 9.0$ kN (upward)

FREE-BODY DIAGRAM OF SEGMENT AD

Point D is 3.0 m from support A.



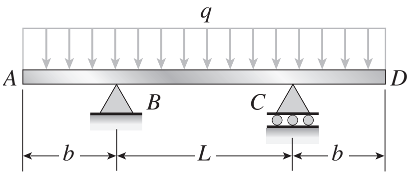
$\Sigma F_{VERT} = 0$: $V = -R_A = -1.0 \text{ kN}$ ←

$\Sigma M_D = 0$: $M = -R_A(3.0 \text{ m}) - 4.0 \text{ kN} \cdot \text{m}$

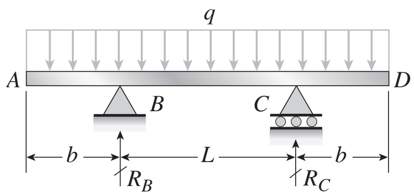
$= -7.0 \text{ kN} \cdot \text{m}$ ←

Problem 4.3-7 The beam *ABCD* shown in the figure has overhangs at each end and carries a uniform load of intensity q.

For what ratio b/L will the bending moment at the midpoint of the beam be zero?



Solution 4.3-7 **Beam with overhangs**

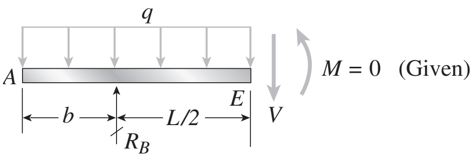


From symmetry and equilibrium of vertical forces:

$$R_B = R_C = q\left(b + \frac{L}{2}\right)$$

FREE-BODY DIAGRAM OF LEFT-HAND HALF OF BEAM:

Point E is at the midpoint of the beam.



$$\Sigma M_E = 0$$

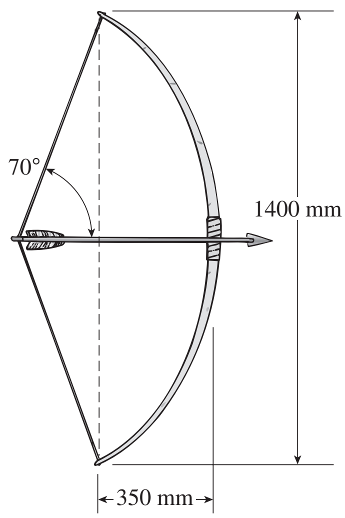
$$-R_B\left(\frac{L}{2}\right) + q\left(\frac{1}{2}\right)\left(b + \frac{L}{2}\right)^2 = 0$$

$$-q\left(b + \frac{L}{2}\right)\left(\frac{L}{2}\right) + q\left(\frac{1}{2}\right)\left(b + \frac{L}{2}\right)^2 = 0$$

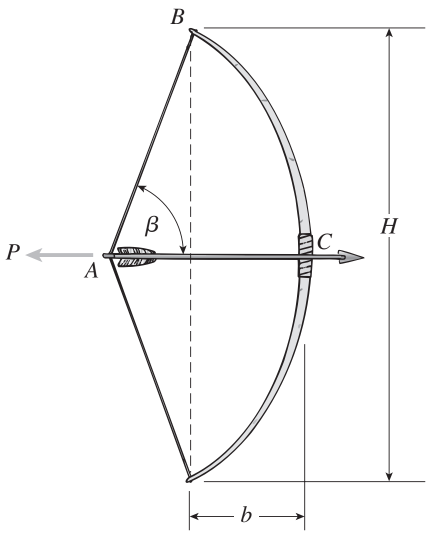
Solve for b/L:

$$\frac{b}{L} = \frac{1}{2} \quad \longleftarrow$$

Problem 4.3-8 At full draw, an archer applies a pull of 130 N to the bowstring of the bow shown in the figure. Determine the bending moment at the midpoint of the bow.



Solution 4.3-8 Archer's bow



$P = 130\,\text{N}$

$\beta = 70°$

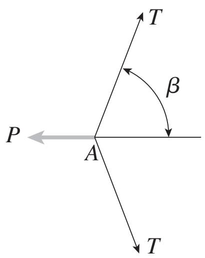
$H = 1400\,\text{mm}$

$\quad = 1.4\,\text{m}$

$b = 350\,\text{mm}$

$\quad = 0.35\,\text{m}$

Free-body diagram of point *A*

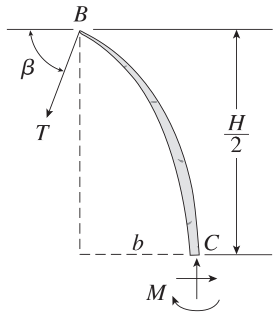


T = tensile force in the bowstring

$\Sigma F_{\text{HORIZ}} = 0\text{:} \quad 2T\cos\beta - P = 0$

$$T = \frac{P}{2\cos\beta}$$

Free-body diagram of segment *BC*



$\Sigma M_C = 0$

$$T(\cos\beta)\left(\frac{H}{2}\right) + T(\sin\beta)(b) - M = 0$$

$$M = T\left(\frac{H}{2}\cos\beta + b\sin\beta\right)$$

$$= \frac{P}{2}\left(\frac{H}{2} + b\tan\beta\right)$$

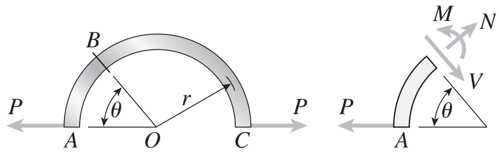
Substitute numerical values:

$$M = \frac{130\text{ N}}{2}\left[\frac{1.4\text{ m}}{2} + (0.35\text{ m})(\tan 70°)\right]$$

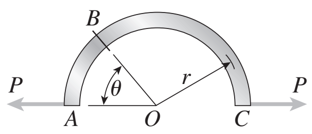
$M = 108\text{ N}\cdot\text{m}$ ←

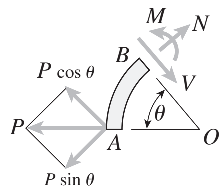
Problem 4.3-9 A curved bar *ABC* is subjected to loads in the form of two equal and opposite forces *P*, as shown in the figure. The axis of the bar forms a semicircle of radius *r*.

Determine the axial force *N*, shear force *V*, and bending moment *M* acting at a cross section defined by the angle θ.



Solution 4.3-9 Curved bar





$\Sigma F_N = 0$ $\nearrow_+ \swarrow^-$ $N - P\sin\theta = 0$

$N = P\sin\theta$ ⟵

$\Sigma F_V = 0$ $\searrow^+ \nwarrow^-$ $V - P\cos\theta = 0$

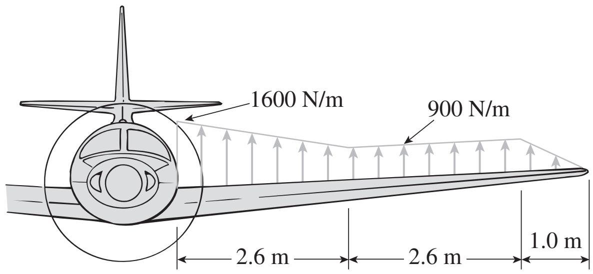
$V = P\cos\theta$ ⟵

$\Sigma M_O = 0$ ↶+ ↷− $M - Nr = 0$

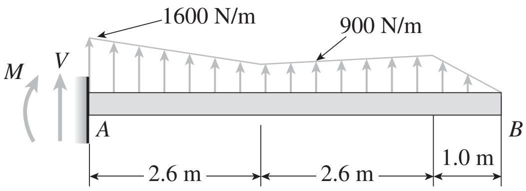
$M = Nr = Pr\sin\theta$ ⟵

Problem 4.3-10 Under cruising conditions the distributed load acting on the wing of a small airplane has the idealized variation shown in the figure.

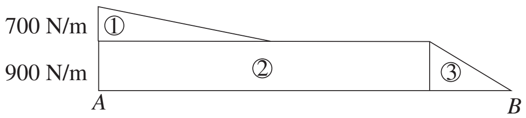
Calculate the shear force *V* and bending moment *M* at the inboard end of the wing.



Solution 4.3-10 Airplane wing



LOADING (IN THREE PARTS)



SHEAR FORCE

$\Sigma F_{VERT} = 0$ $\uparrow_+ \downarrow^-$

$$V + \frac{1}{2}(700\text{ N/m})(2.6\text{ m}) + (900\text{ N/m})(5.2\text{ m}) + \frac{1}{2}(900\text{ N/m})(1.0\text{ m}) = 0$$

$V = -6040\text{ N} = -6.04\text{ kN}$ ⟵

(Minus means the shear force acts opposite to the direction shown in the figure.)

BENDING MOMENT

$\Sigma M_A = 0$ ↶+ ↷−

$$-M + \frac{1}{2}(700\text{ N/m})(2.6\text{ m})\left(\frac{2.6\text{ m}}{3}\right) + (900\text{ N/m})(5.2\text{ m})(2.6\text{ m}) + \frac{1}{2}(900\text{ N/m})(1.0\text{ m})\left(5.2\text{ m} + \frac{1.0\text{ m}}{3}\right) = 0$$

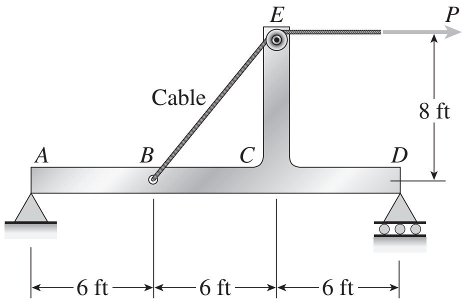
$M = 788.67\text{ N}\cdot\text{m} + 12{,}168\text{ N}\cdot\text{m} + 2490\text{ N}\cdot\text{m}$

$= 15{,}450\text{ N}\cdot\text{m}$

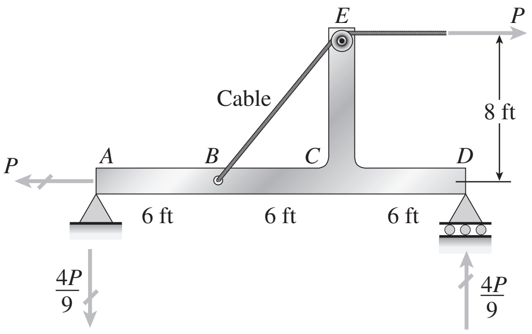
$= 15.45\text{ kN}\cdot\text{m}$ ⟵

Problem 4.3-11 A beam *ABCD* with a vertical arm *CE* is supported as a simple beam at *A* and *D* (see figure). A cable passes over a small pulley that is attached to the arm at *E*. One end of the cable is attached to the beam at point *B*.

What is the force *P* in the cable if the bending moment in the beam just to the left of point *C* is equal numerically to 640 lb-ft? (*Note:* Disregard the widths of the beam and vertical arm and use centerline dimensions when making calculations.)



Solution 4.3-11 **Beam with a cable**

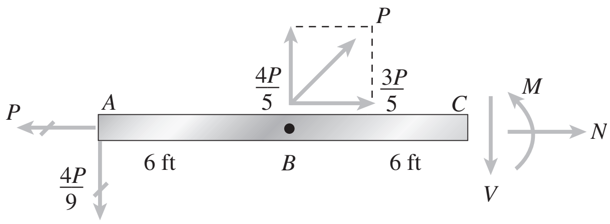


UNITS:

P in lb

M in lb-ft

FREE-BODY DIAGRAM OF SECTION *AC*



$\Sigma M_C = 0$ ⊕↶

$$M - \frac{4P}{5}(6\text{ ft}) + \frac{4P}{9}(12\text{ ft}) = 0$$

$$M = -\frac{8P}{15}\text{ lb-ft}$$

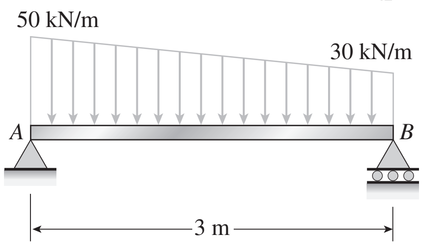
Numerical value of *M* equals 640 lb-ft.

$$\therefore\ 640\text{ lb-ft} = \frac{8P}{15}\text{ lb-ft}$$

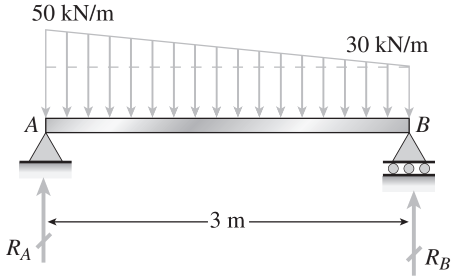
and $P = 1200$ lb ←

Problem 4.3-12 A simply supported beam *AB* supports a trapezoidally distributed load (see figure). The intensity of the load varies linearly from 50 kN/m at support *A* to 30 kN/m at support *B*.

Calculate the shear force *V* and bending moment *M* at the midpoint of the beam.

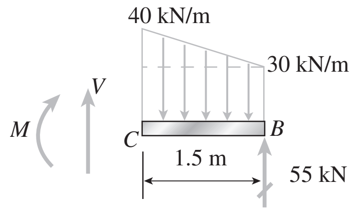


Solution 4.3-12 Beam with trapezoidal load



FREE-BODY DIAGRAM OF SECTION *CB*

Point *C* is at the midpoint of the beam.



REACTIONS

$\Sigma M_B = 0 \quad -R_A(3\text{ m}) + (30\text{ kN/m})(3\text{ m})(1.5\text{ m})$

$+ (20\text{ kN/m})(3\text{ m})(1/2)(2\text{ m}) = 0$

$R_A = 65\text{ kN}$

$\Sigma F_{VERT} = 0 \quad \overset{+}{\uparrow}$

$R_A + R_B - 1/2(50\text{ kN/m} + 30\text{ kN/m})(3\text{ m}) = 0$

$R_B = 55\text{ kN}$

$\Sigma F_{VERT} = 0 \quad \uparrow_+ \downarrow^-$

$V - (30\text{ kN/m})(1.5\text{ m}) - \frac{1}{2}(10\text{ kN/m})(1.5\text{ m})$

$+ 55\text{ kN} = 0$

$V = -2.5\text{ kN}$ ←

$\Sigma M_C = 0$

$-M - (30\text{ kN/m})(1.5\text{ m})(0.75\text{ m})$

$-1/2(10\text{ kN/m})(1.5\text{ m})(0.5\text{ m})$

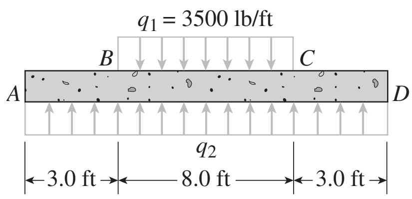
$+ (55\text{ kN})(1.5\text{ m}) = 0$

$M = 45.0\text{ kN}\cdot\text{m}$ ←

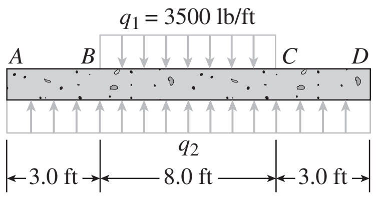
Problem 4.3-13 Beam *ABCD* represents a reinforced-concrete foundation beam that supports a uniform load of intensity $q_1 = 3500$ lb/ft (see figure). Assume that the soil pressure on the underside of the beam is uniformly distributed with intensity q_2.

(a) Find the shear force V_B and bending moment M_B at point *B*.

(b) Find the shear force V_m and bending moment M_m at the midpoint of the beam.



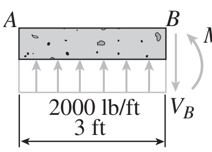
Solution 4.3-13 Foundation beam



$\Sigma F_{VERT} = 0: \quad q_2(14\text{ ft}) = q_1(8\text{ ft})$

$\therefore q_2 = \frac{8}{14} q_1 = 2000\text{ lb/ft}$

(a) *V* AND *M* AT POINT *B*

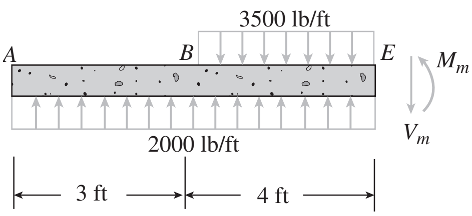


$\Sigma F_{VERT} = 0:$

$V_B = 6000\text{ lb}$ ←

$\Sigma M_B = 0: \quad M_B = 9000\text{ lb-ft}$ ←

(b) *V* AND *M* AT MIDPOINT *E*



$\Sigma F_{VERT} = 0: V_m = (2000\text{ lb/ft})(7\text{ ft}) - (3500\text{ lb/ft})(4\text{ ft})$

$V_m = 0$ ←

$\Sigma M_E = 0:$

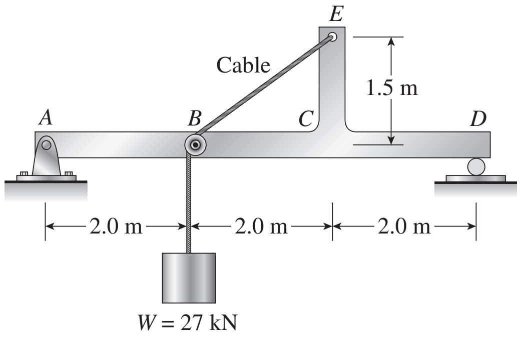
$M_m = (2000\text{ lb/ft})(7\text{ ft})(3.5\text{ ft})$

$- (3500\text{ lb/ft})(4\text{ ft})(2\text{ ft})$

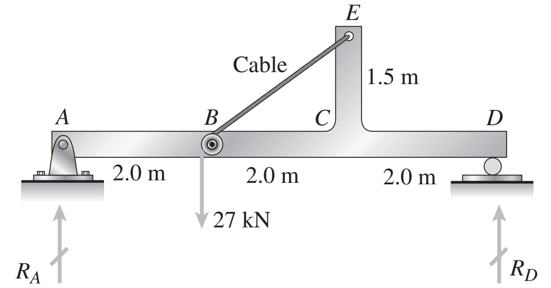
$M_m = 21{,}000\text{ lb-ft}$ ←

Problem 4.3-14 The simply-supported beam *ABCD* is loaded by a weight W = 27 kN through the arrangement shown in the figure. The cable passes over a small frictionless pulley at *B* and is attached at *E* to the end of the vertical arm.

Calculate the axial force *N*, shear force *V*, and bending moment M at section C, which is just to the left of the vertical arm. (*Note:* Disregard the widths of the beam and vertical arm and use centerline dimensions when making calculations.)

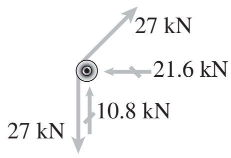


Solution 4.3-14 **Beam with cable and weight**

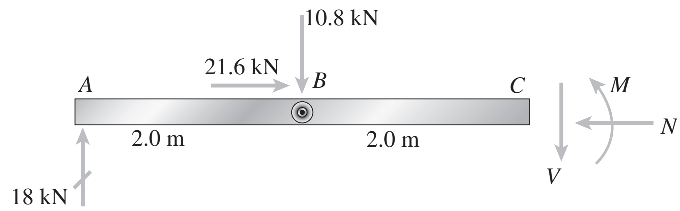


$R_A = 18$ kN $\quad R_D = 9$ kN

FREE-BODY DIAGRAM OF PULLEY AT *B*



FREE-BODY DIAGRAM OF SEGMENT *ABC* OF BEAM



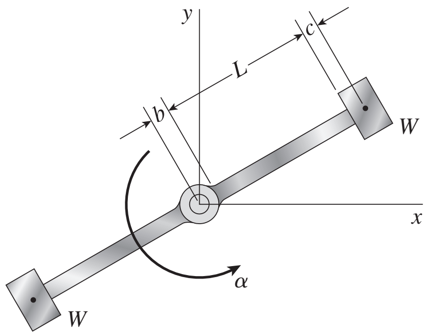
$\Sigma F_{HORIZ} = 0$: $N = 21.6$ kN (compression) ←

$\Sigma F_{VERT} = 0$: $V = 7.2$ kN ←

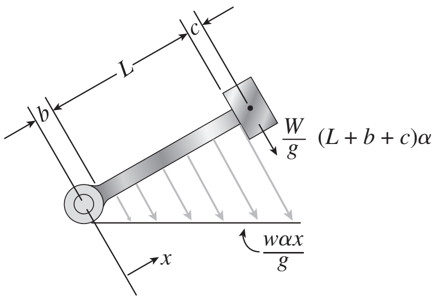
$\Sigma M_C = 0$: $M = 50.4$ kN · m ←

Problem 4.3-15 The centrifuge shown in the figure rotates in a horizontal plane (the *xy* plane) on a smooth surface about the *z* axis (which is vertical) with an angular acceleration α. Each of the two arms has weight *w* per unit length and supports a weight $W = 2.0\, wL$ at its end.

Derive formulas for the maximum shear force and maximum bending moment in the arms, assuming $b = L/9$ and $c = L/10$.



Solution 4.3-15 **Rotating centrifuge**



Tangential acceleration $= r\alpha$

Inertial force $Mr\alpha = \frac{W}{g} r\alpha$

Maximum *V* and *M* occur at $x = b$.

$$V_{max} = \frac{W}{g}(L + b + c)\alpha + \int_b^{L+b} \frac{w\alpha}{g} x\, dx$$
$$= \frac{W\alpha}{g}(L + b + c) + \frac{wL\alpha}{2g}(L + 2b) \longleftarrow$$

$$M_{max} = \frac{W\alpha}{g}(L + b + c)(L + c) + \int_b^{L+b} \frac{w\alpha}{g} x(x - b)dx$$
$$= \frac{W\alpha}{g}(L + b + c)(L + c) + \frac{wL^2\alpha}{6g}(2L + 3b) \longleftarrow$$

Substitute numerical data:

$$W = 2.0\, wL \quad b = \frac{L}{9} \quad c = \frac{L}{10}$$

$$V_{max} = \frac{91wL^2\alpha}{30g} \longleftarrow$$

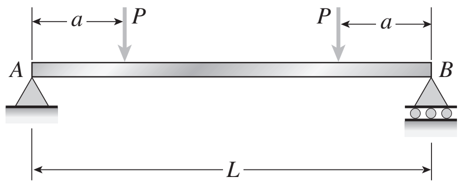
$$M_{max} = \frac{229wL^3\alpha}{75g} \longleftarrow$$

Shear-Force and Bending-Moment Diagrams

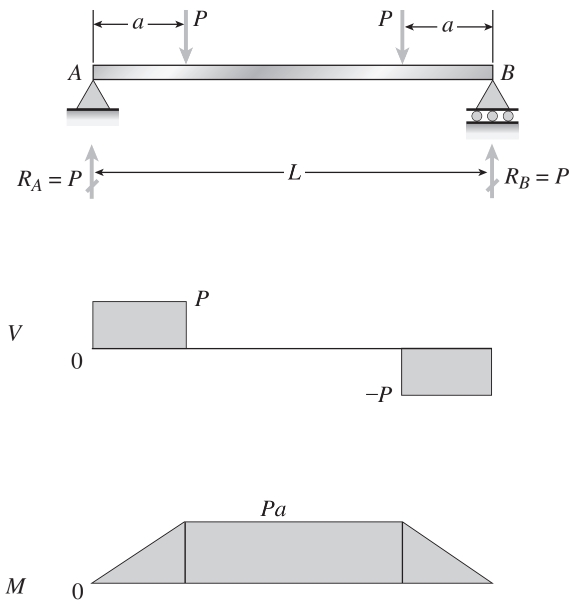
When solving the problems for Section 4.5, draw the shear-force and bending-moment diagrams approximately to scale and label all critical ordinates, including the maximum and minimum values.

Probs. 4.5-1 through 4.5-10 are symbolic problems and Probs. 4.5-11 through 4.5-24 are numerical problems. The remaining problems (4.5-25 through 4.5-30) involve specialized topics, such as optimization, beams with hinges, and moving loads.

Problem 4.5-1 Draw the shear-force and bending-moment diagrams for a simple beam *AB* supporting two equal concentrated loads *P* (see figure).

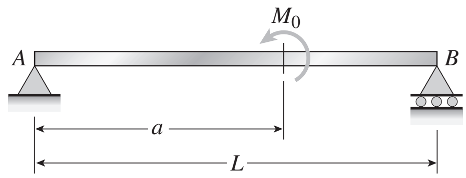


Solution 4.5-1 Simple beam

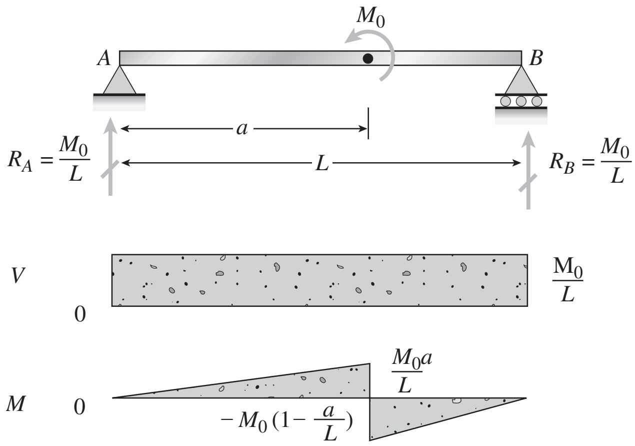


Problem 4.5-2 A simple beam *AB* is subjected to a counterclockwise couple of moment M_0 acting at distance *a* from the left-hand support (see figure).

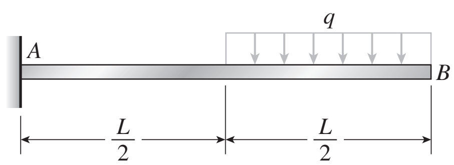
Draw the shear-force and bending-moment diagrams for this beam.



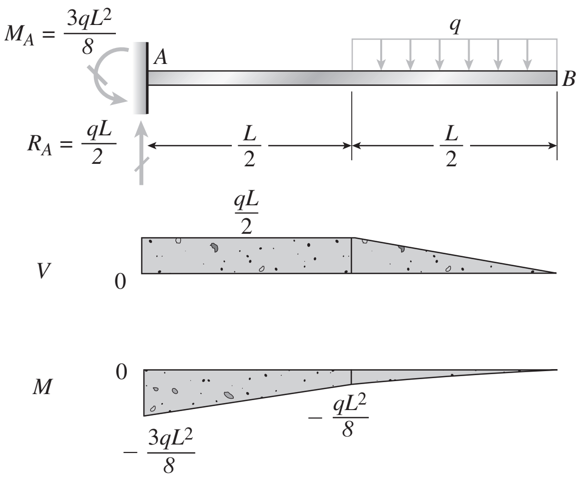
Solution 4.5-2 **Simple beam**



Problem 4.5-3 Draw the shear-force and bending-moment diagrams for a cantilever beam *AB* carrying a uniform load of intensity *q* over one-half of its length (see figure).

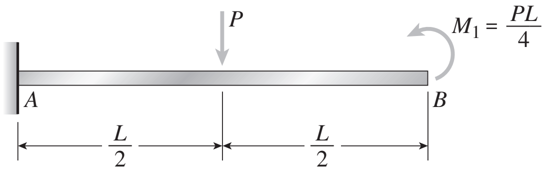


Solution 4.5-3 **Cantilever beam**

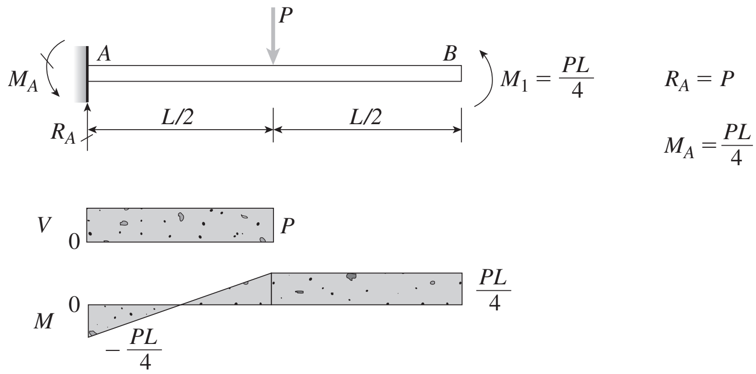


Problem 4.5-4 The cantilever beam *AB* shown in the figure is subjected to a concentrated load *P* at the midpoint and a counterclockwise couple of moment $M_1 = PL/4$ at the free end.

Draw the shear-force and bending-moment diagrams for this beam.

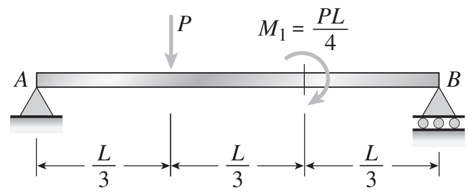


Solution 4.5-4 Cantilever beam

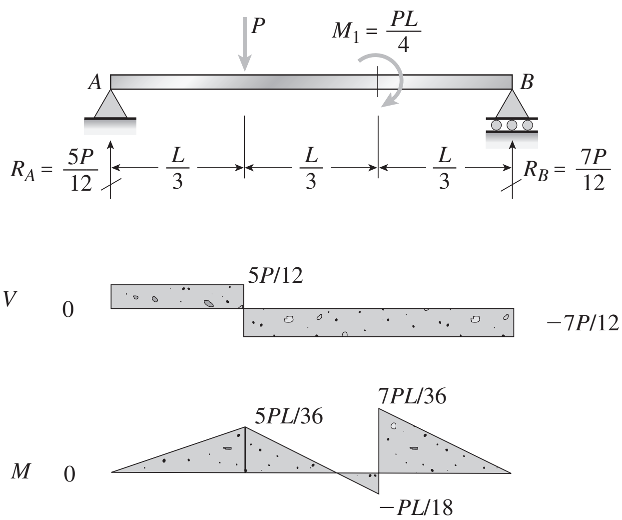


Problem 4.5-5 The simple beam *AB* shown in the figure is subjected to a concentrated load *P* and a clockwise couple $M_1 = PL/4$ acting at the third points.

Draw the shear-force and bending-moment diagrams for this beam.

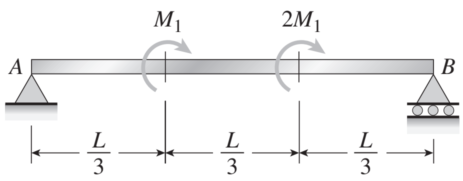


Solution 4.5-5 Simple beam

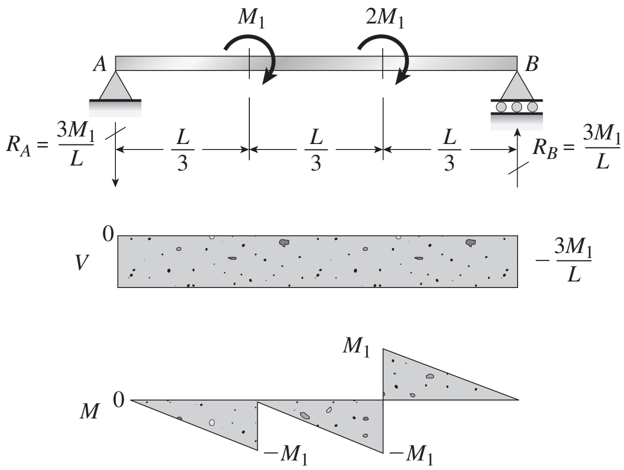


Problem 4.5-6 A simple beam AB subjected to clockwise couples M_1 and $2M_1$ acting at the third points is shown in the figure.

Draw the shear-force and bending-moment diagrams for this beam.

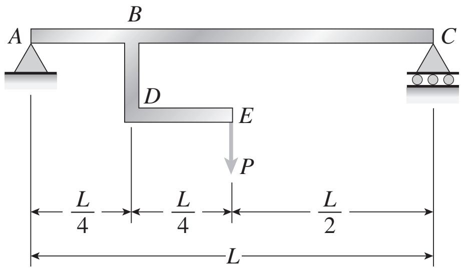


Solution 4.5-6 **Simple beam**

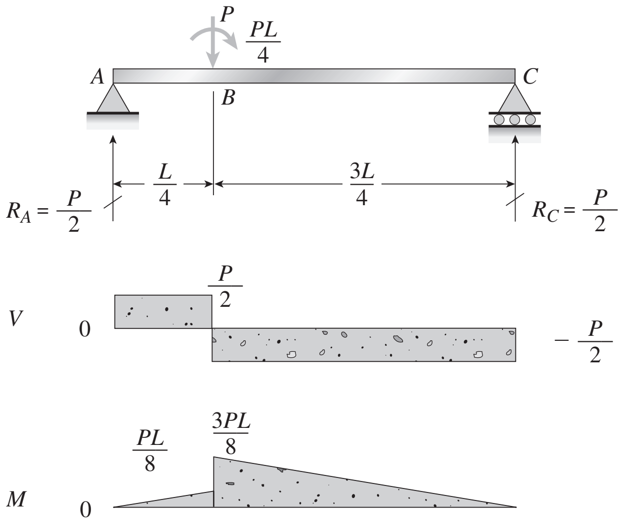


Problem 4.5-7 A simply supported beam ABC is loaded by a vertical load P acting at the end of a bracket BDE (see figure).

Draw the shear-force and bending-moment diagrams for beam ABC.

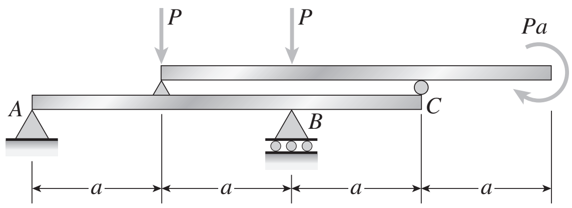


Solution 4.5-7 **Beam with bracket**

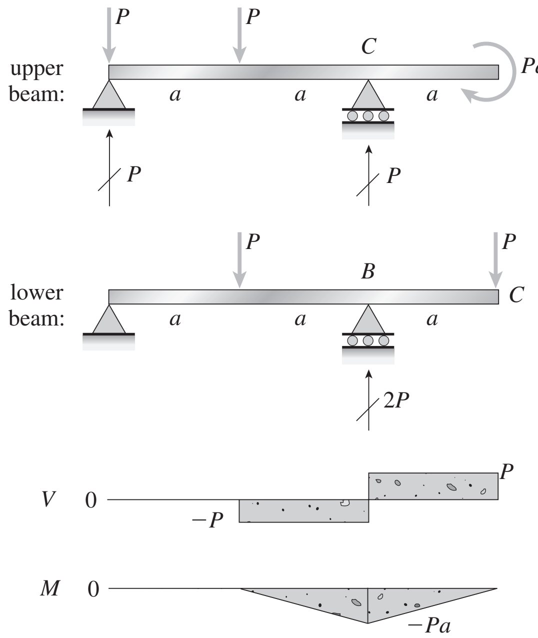


Problem 4.5-8 A beam *ABC* is simply supported at *A* and *B* and has an overhang *BC* (see figure). The beam is loaded by two forces *P* and a clockwise couple of moment *Pa* that act through the arrangement shown.

Draw the shear-force and bending-moment diagrams for beam *ABC*.

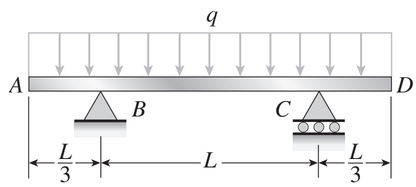


Solution 4.5-8 Beam with overhang

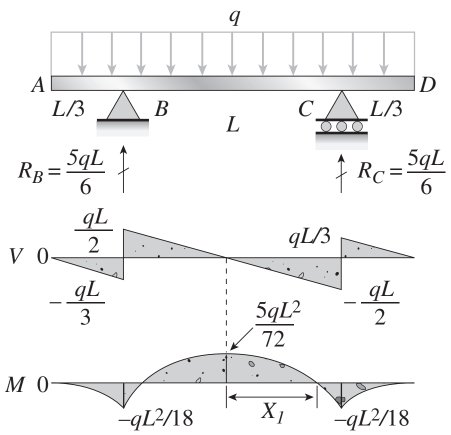


Problem 4.5-9 Beam *ABCD* is simply supported at *B* and *C* and has overhangs at each end (see figure). The span length is *L* and each overhang has length $L/3$. A uniform load of intensity q acts along the entire length of the beam.

Draw the shear-force and bending-moment diagrams for this beam.

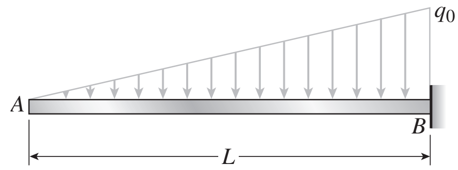


Solution 4.5-9 Beam with overhangs

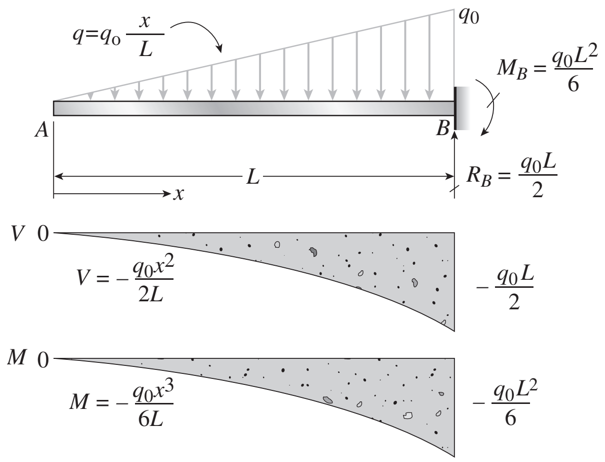


$$x_1 = L\frac{\sqrt{5}}{6} = 0.3727L$$

Problem 4.5-10 Draw the shear-force and bending-moment diagrams for a cantilever beam *AB* supporting a linearly varying load of maximum intensity q_0 (see figure).

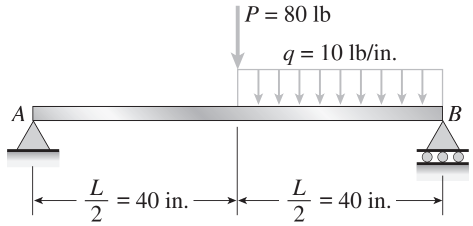


Solution 4.5-10 Cantilever beam

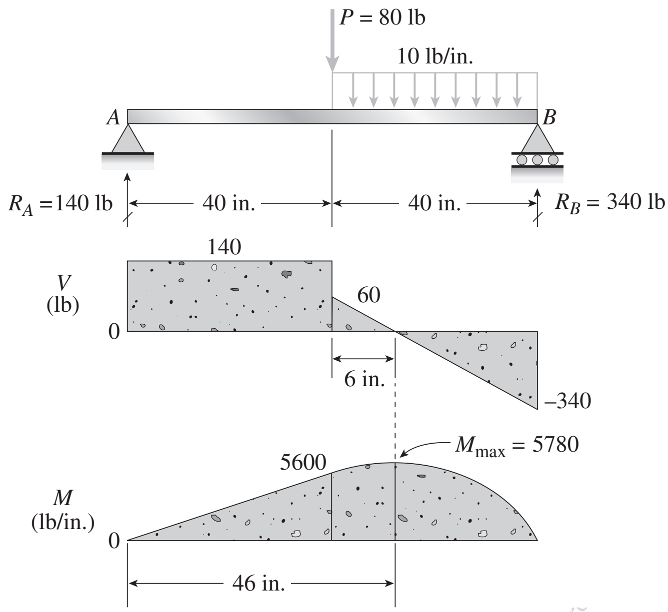


Problem 4.5-11 The simple beam *AB* supports a uniform load of intensity $q = 10$ lb/in. acting over one-half of the span and a concentrated load $P = 80$ lb acting at midspan (see figure).

Draw the shear-force and bending-moment diagrams for this beam.

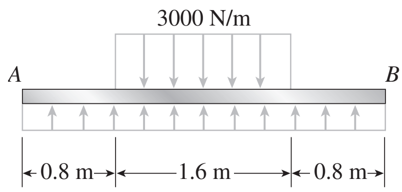


Solution 4.5-11 Simple beam

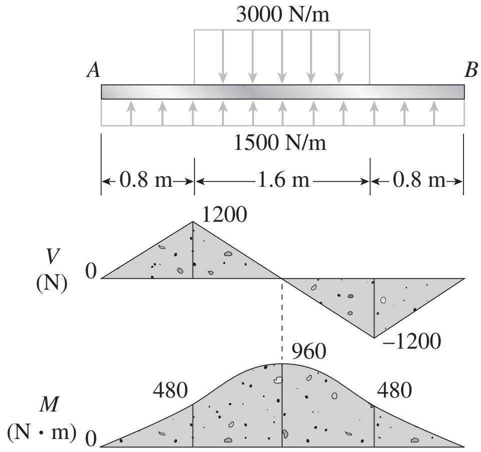


Problem 4.5-12 The beam *AB* shown in the figure supports a uniform load of intensity 3000 N/m acting over half the length of the beam. The beam rests on a foundation that produces a uniformly distributed load over the entire length.

Draw the shear-force and bending-moment diagrams for this beam.

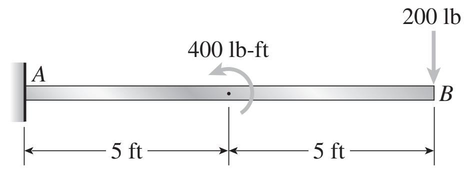


Solution 4.5-12 Beam with distributed loads

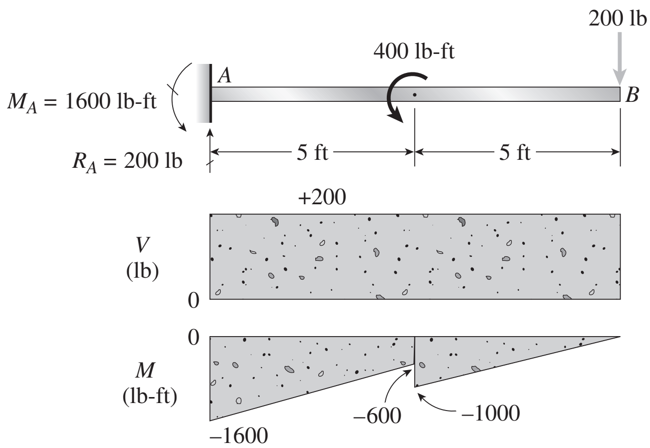


Problem 4.5-13 A cantilever beam *AB* supports a couple and a concentrated load, as shown in the figure.

Draw the shear-force and bending-moment diagrams for this beam.

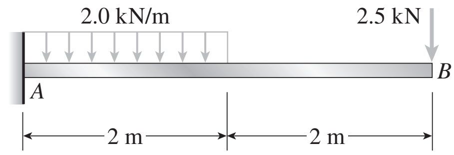


Solution 4.5-13 Cantilever beam

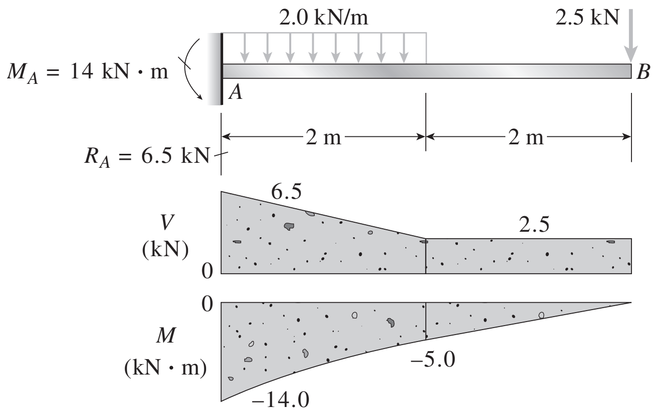


Problem 4.5-14 The cantilever beam *AB* shown in the figure is subjected to a uniform load acting throughout one-half of its length and a concentrated load acting at the free end.

Draw the shear-force and bending-moment diagrams for this beam.

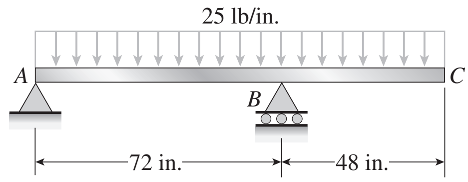


Solution 4.5-14 **Cantilever beam**

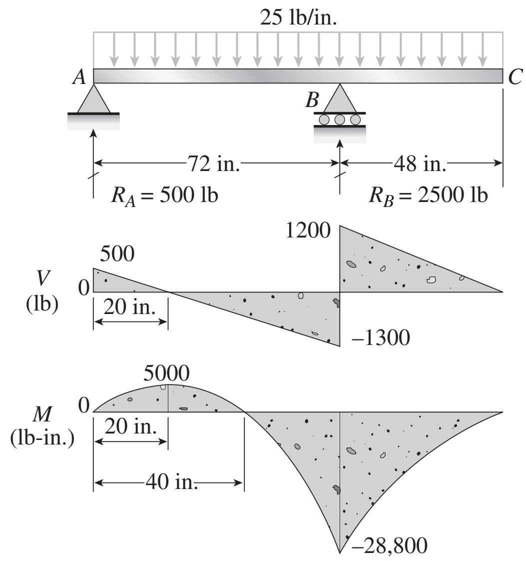


Problem 4.5-15 The uniformly loaded beam *ABC* has simple supports at *A* and *B* and an overhang *BC* (see figure).

Draw the shear-force and bending-moment diagrams for this beam.

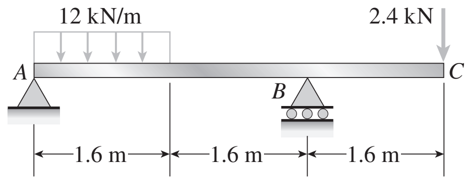


Solution 4.5-15 **Beam with an overhang**

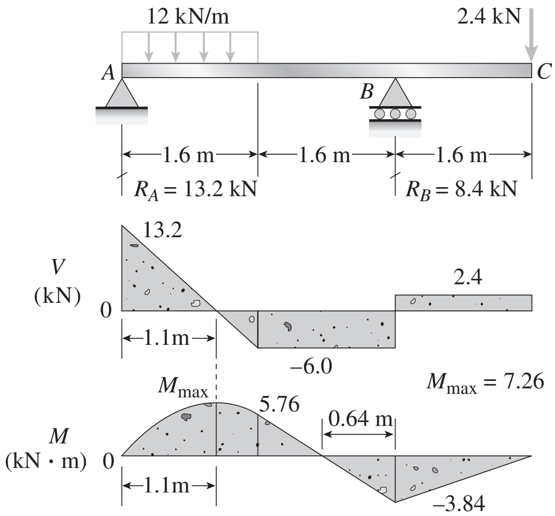


Problem 4.5-16 A beam ABC with an overhang at one end supports a uniform load of intensity 12 kN/m and a concentrated load of magnitude 2.4 kN (see figure).

Draw the shear-force and bending-moment diagrams for this beam.

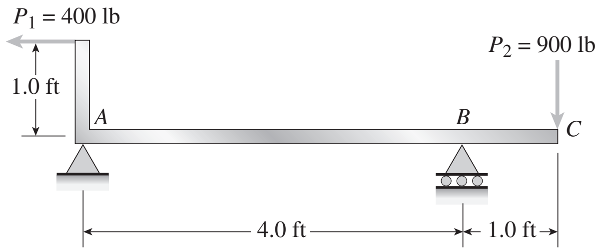


Solution 4.5-16 **Beam with an overhang**

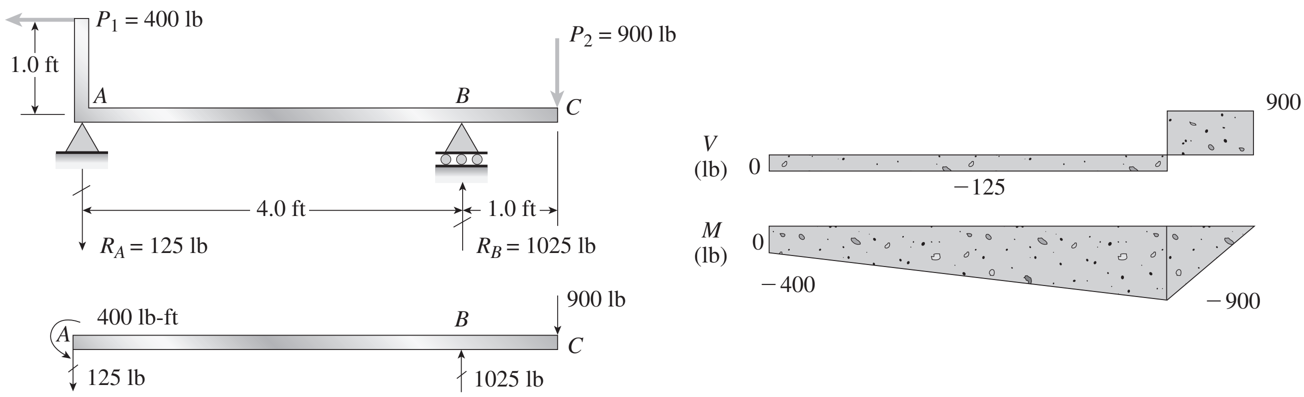


Problem 4.5-17 The beam ABC shown in the figure is simply supported at A and B and has an overhang from B to C. The loads consist of a horizontal force P_1 = 400 lb acting at the end of the vertical arm and a vertical force P_2 = 900 lb acting at the end of the overhang.

Draw the shear-force and bending-moment diagrams for this beam. (*Note:* Disregard the widths of the beam and vertical arm and use centerline dimensions when making calculations.)

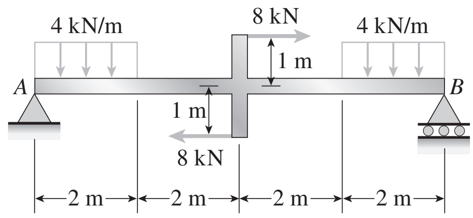


Solution 4.5-17 **Beam with vertical arm**

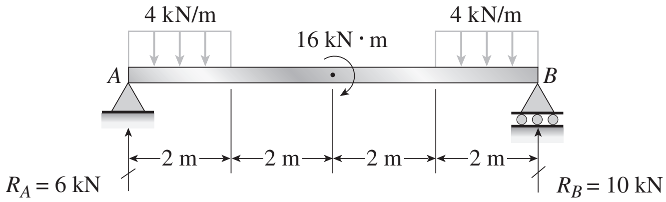


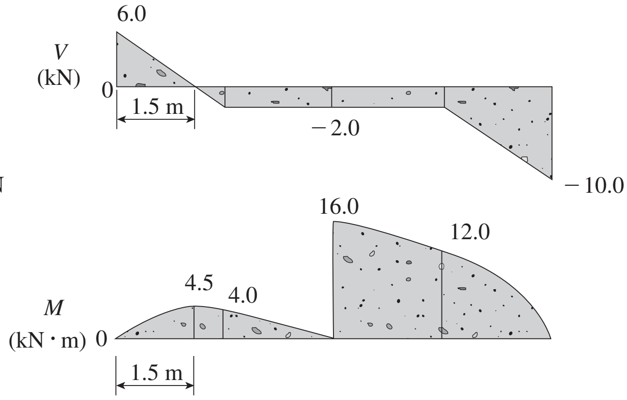
Problem 4.5-18 A simple beam *AB* is loaded by two segments of uniform load and two horizontal forces acting at the ends of a vertical arm (see figure).

Draw the shear-force and bending-moment diagrams for this beam.



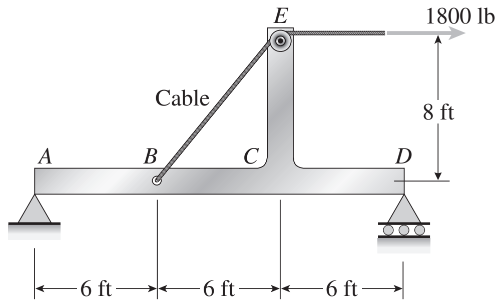
Solution 4.5-18 **Simple beam**



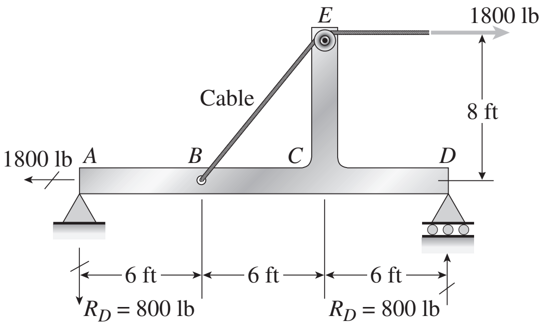


Problem 4.5-19 A beam *ABCD* with a vertical arm *CE* is supported as a simple beam at *A* and *D* (see figure). A cable passes over a small pulley that is attached to the arm at *E*. One end of the cable is attached to the beam at point *B*. The tensile force in the cable is 1800 lb.

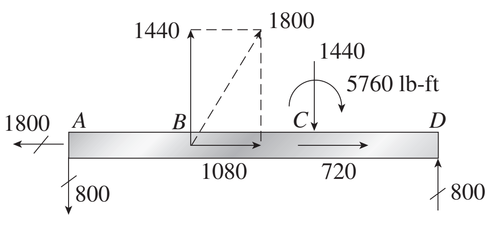
Draw the shear-force and bending-moment diagrams for beam *ABCD*. (*Note:* Disregard the widths of the beam and vertical arm and use center-line dimensions when making calculations.)



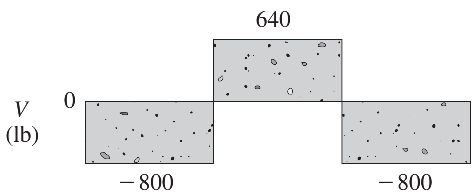
Solution 4.5-19 **Beam with a cable**

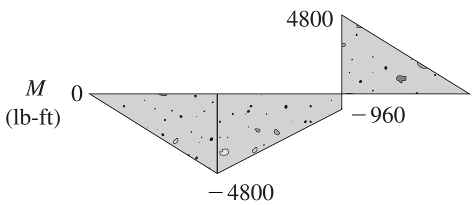


FREE-BODY DIAGRAM OF BEAM *ABCD*



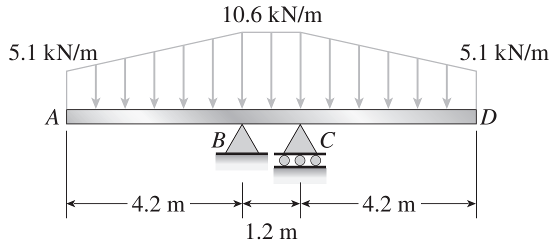
Note: All forces have units of pounds.



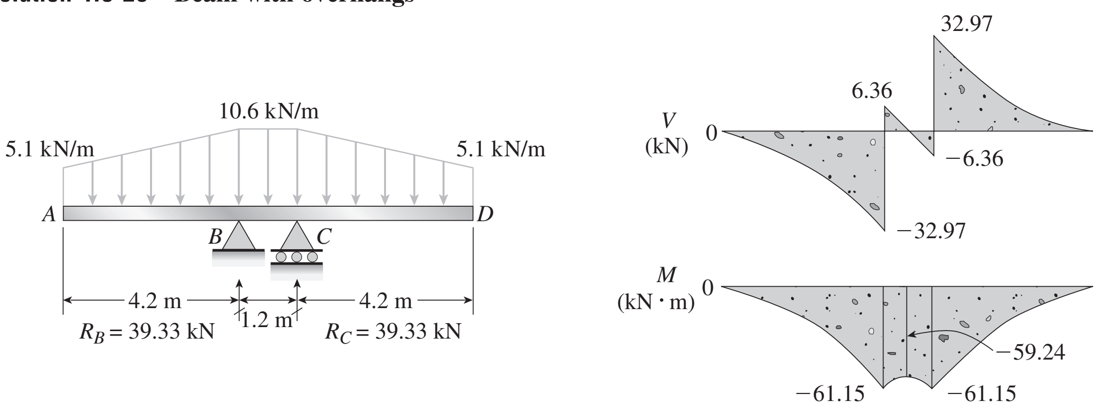


Problem 4.5-20 The beam *ABCD* shown in the figure has overhangs that extend in both directions for a distance of 4.2 m from the supports at *B* and *C*, which are 1.2 m apart.

Draw the shear-force and bending-moment diagrams for this overhanging beam.

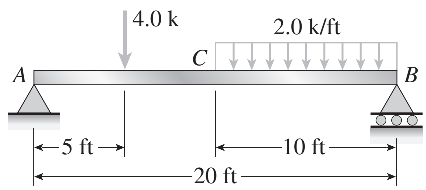


Solution 4.5-20 **Beam with overhangs**

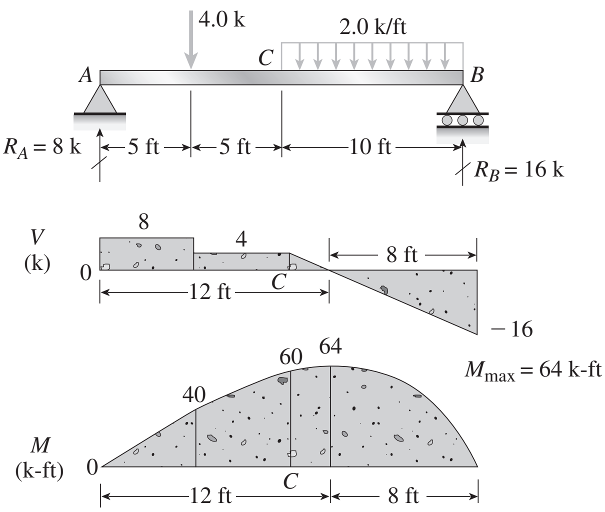


Problem 4.5-21 The simple beam *AB* shown in the figure supports a concentrated load and a segment of uniform load.

Draw the shear-force and bending-moment diagrams for this beam.

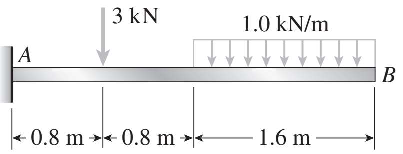


Solution 4.5-21 **Simple beam**

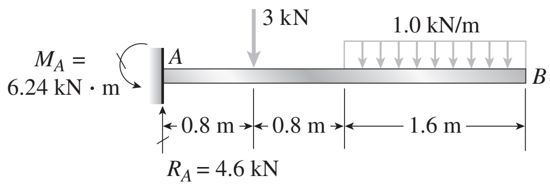


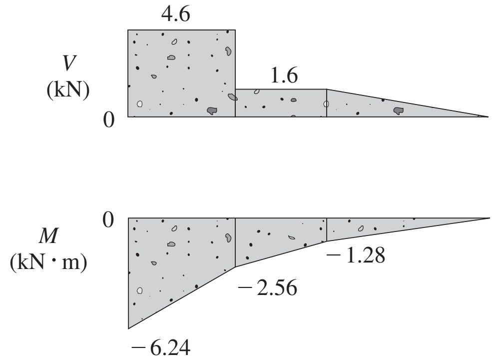
Problem 4.5-22 The cantilever beam shown in the figure supports a concentrated load and a segment of uniform load.

Draw the shear-force and bending-moment diagrams for this cantilever beam.



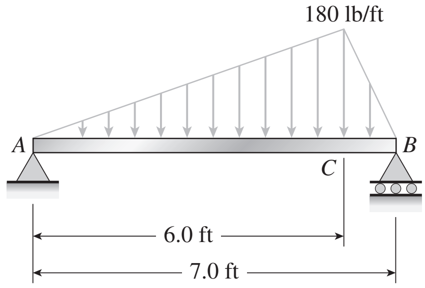
Solution 4.5-22 **Cantilever beam**



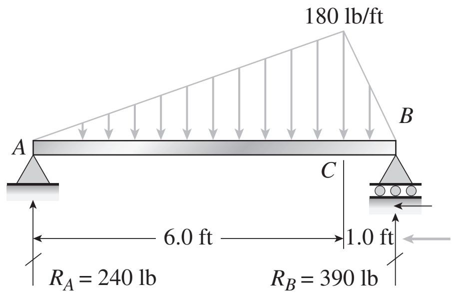


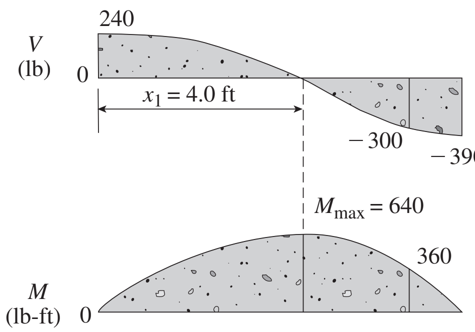
Problem 4.5-23 The simple beam *ACB* shown in the figure is subjected to a triangular load of maximum intensity 180 lb/ft.

Draw the shear-force and bending-moment diagrams for this beam.



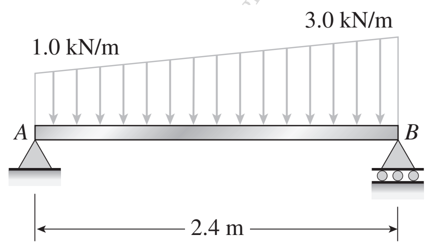
Solution 4.5-23 **Simple beam**



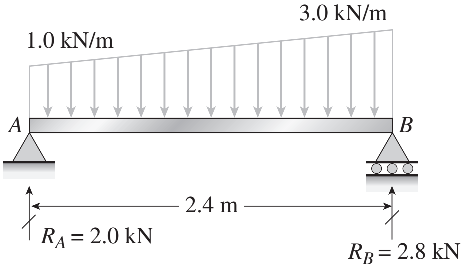


Problem 4.5-24 A beam with simple supports is subjected to a trapezoidally distributed load (see figure). The intensity of the load varies from 1.0 kN/m at support *A* to 3.0 kN/m at support *B*.

Draw the shear-force and bending-moment diagrams for this beam.

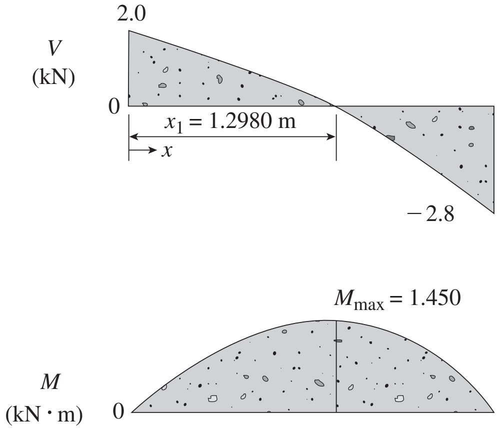


Solution 4.5-24 **Simple beam**



$$V = 2.0 - x - \frac{x^2}{2.4} \quad (x = \text{meters};\ V = \text{kN})$$

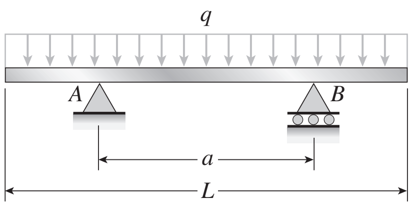
Set $V = 0$: $x_1 = 1.2980\,\text{m}$



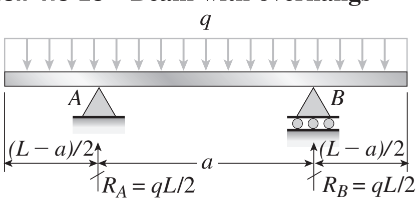
Problem 4.5-25 A beam of length L is being designed to support a uniform load of intensity q (see figure). If the supports of the beam are placed at the ends, creating a simple beam, the maximum bending moment in the beam is $qL^2/8$. However, if the supports of the beam are moved symmetrically toward the middle of the beam (as pictured), the maximum bending moment is reduced.

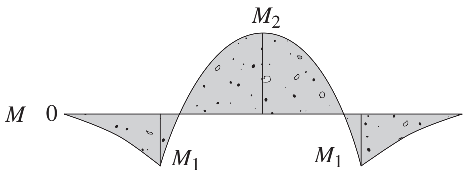
Determine the distance a between the supports so that the maximum bending moment in the beam has the smallest possible numerical value.

Draw the shear-force and bending-moment diagrams for this condition.



Solution 4.5-25 **Beam with overhangs**





The maximum bending moment is smallest when $M_1 = M_2$ (numerically).

$$M_1 = \frac{q(L-a)^2}{8}$$

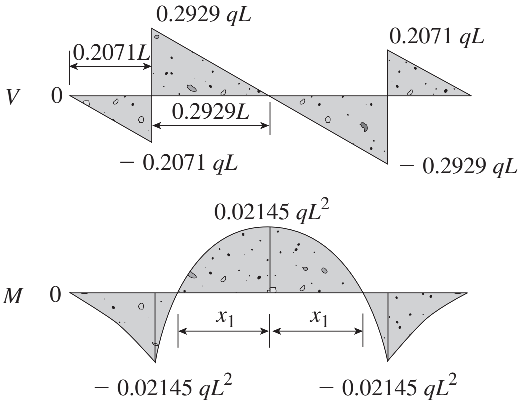
$$M_2 = R_A\left(\frac{a}{2}\right) - \frac{qL^2}{8} = \frac{qL}{8}(2a - L)$$

$$M_1 = M_2 \qquad (L-a)^2 = L(2a - L)$$

Solve for a: $a = (2 - \sqrt{2})L = 0.5858L$ ←

$$M_1 = M_2 = \frac{q}{8}(L-a)^2$$

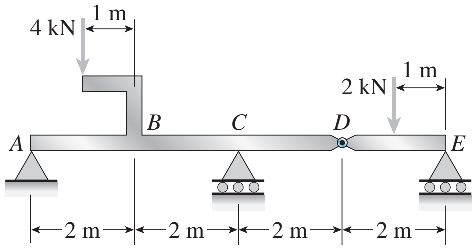
$$= \frac{qL^2}{8}(3 - 2\sqrt{2}) = 0.02145qL^2 \quad \leftarrow$$



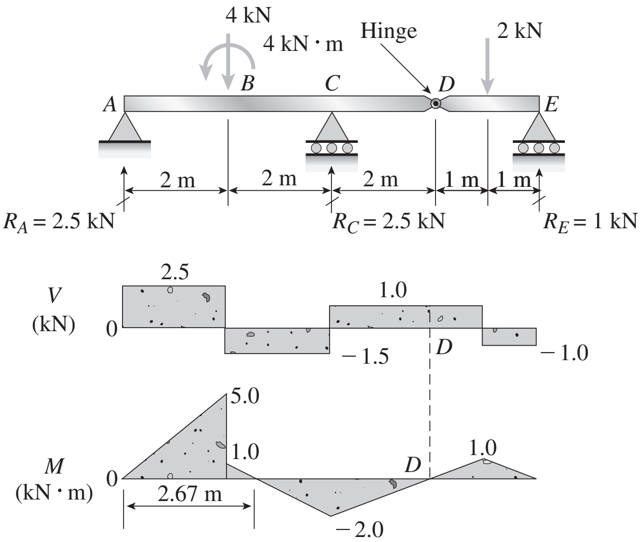
$x_1 = 0.3536\,a$
$= 0.2071\,L$

Problem 4.5-26 The compound beam *ABCDE* shown in the figure consists of two beams (*AD* and *DE*) joined by a hinged connection at *D*. The hinge can transmit a shear force but not a bending moment. The loads on the beam consist of a 4-kN force at the end of a bracket attached at point *B* and a 2-kN force at the midpoint of beam *DE*.

Draw the shear-force and bending-moment diagrams for this compound beam.

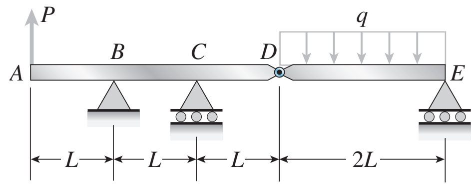


Solution 4.5-26 Compound beam

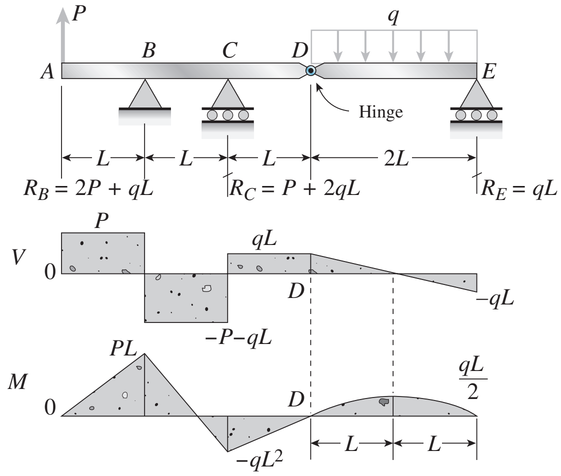


Problem 4.5-27 The compound beam *ABCDE* shown in the figure consists of two beams (*AD* and *DE*) joined by a hinged connection at *D*. The hinge can transmit a shear force but not a bending moment. A force *P* acts upward at *A* and a uniform load of intensity *q* acts downward on beam *DE*.

Draw the shear-force and bending-moment diagrams for this compound beam.

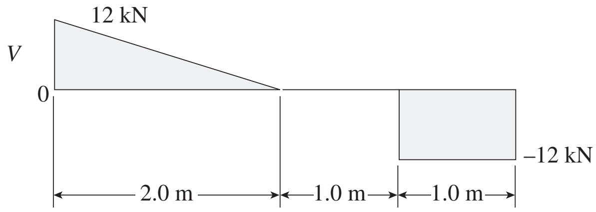


Solution 4.5-27 Compound beam

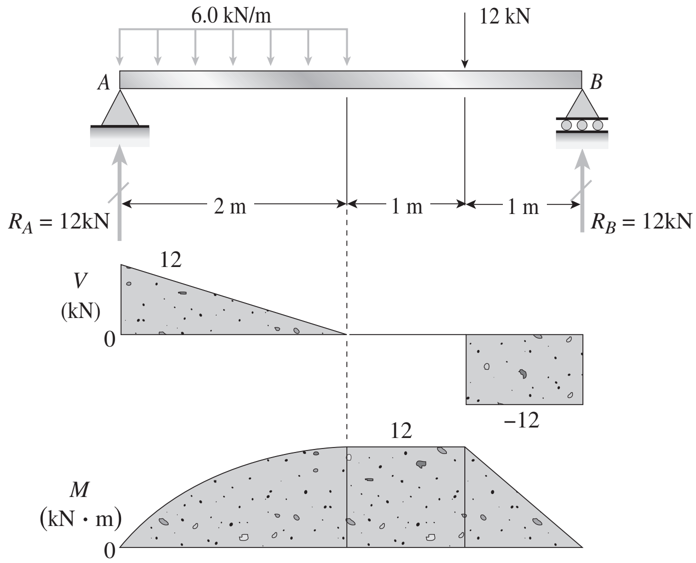


Problem 4.5-28 The shear-force diagram for a simple beam is shown in the figure.

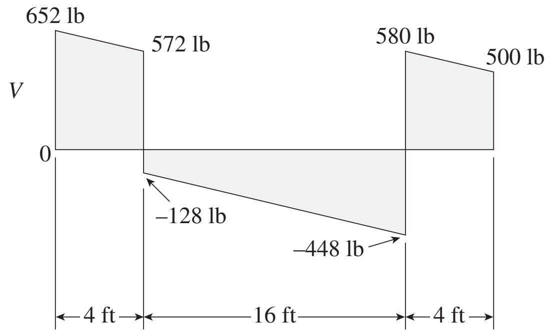
Determine the loading on the beam and draw the bending-moment diagram, assuming that no couples act as loads on the beam.



Solution 4.5-28 **Simple beam (*V* is given)**

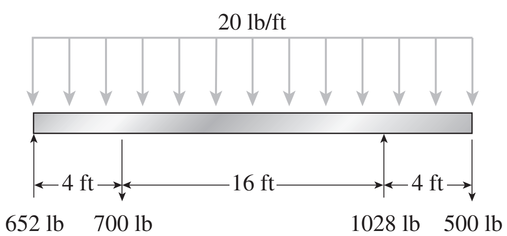


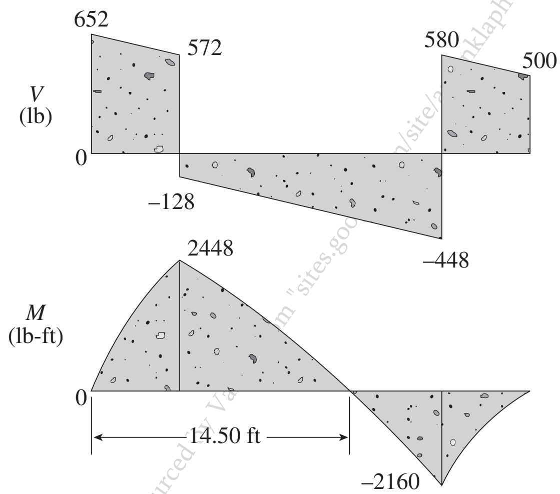
Problem 4.5-29 The shear-force diagram for a beam is shown in the figure. Assuming that no couples act as loads on the beam, determine the forces acting on the beam and draw the bending-moment diagram.



Solution 4.5-29 **Forces on a beam (*V* is given)**

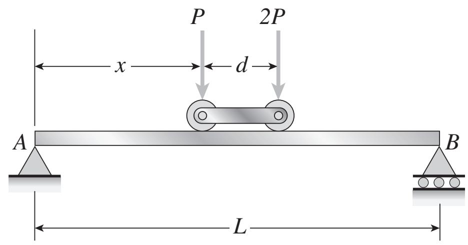
FORCE DIAGRAM



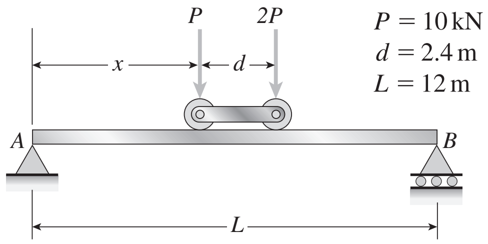


Problem 4.5-30 A simple beam AB supports two connected wheel loads P and $2P$ that are distance d apart (see figure). The wheels may be placed at any distance x from the left-hand support of the beam.

(a) Determine the distance x that will produce the maximum shear force in the beam, and also determine the maximum shear force V_{max}.
(b) Determine the distance x that will produce the maximum bending moment in the beam, and also draw the corresponding bending-moment diagram. (Assume $P = 10$ kN, $d = 2.4$ m, and $L = 12$ m.)



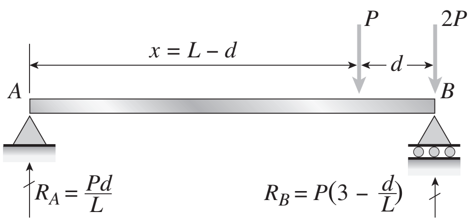
Solution 4.5-30 Moving loads on a beam



$P = 10\,\text{kN}$
$d = 2.4\,\text{m}$
$L = 12\,\text{m}$

(a) MAXIMUM SHEAR FORCE

By inspection, the maximum shear force occurs at support B when the larger load is placed close to, but not directly over, that support.

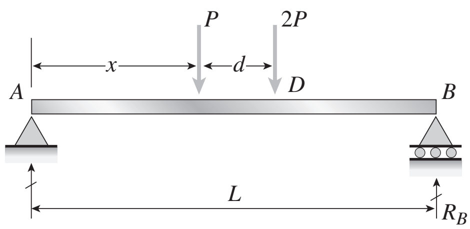


$x = L - d = 9.6$ m ←

$$V_{max} = R_B = P\left(3 - \frac{d}{L}\right) = 28 \text{ kN} \quad \leftarrow$$

(b) MAXIMUM BENDING MOMENT

By inspection, the maximum bending moment occurs at point D, under the larger load $2P$.



Reaction at support B:

$$R_B = \frac{P}{L}x + \frac{2P}{L}(x + d) = \frac{P}{L}(2d + 3x)$$

Bending moment at D:

$$M_D = R_B(L - x - d)$$
$$= \frac{P}{L}(2d + 3x)(L - x - d)$$
$$= \frac{P}{L}[-3x^2 + (3L - 5d)x + 2d(L - d)] \qquad \text{Eq.(1)}$$

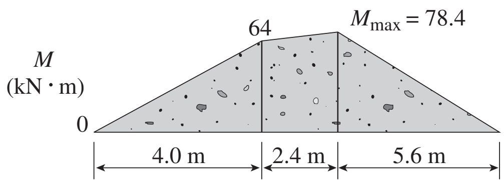
$$\frac{dM_D}{dx} = \frac{P}{L}(-6x + 3L - 5d) = 0$$

Solve for x: $x = \dfrac{L}{6}\left(3 - \dfrac{5d}{L}\right) = 4.0$ m ←

Substitute x into Eq (1):

$$M_{max} = \frac{P}{L}\left[-3\left(\frac{L}{6}\right)^2\left(3 - \frac{5d}{L}\right)^2 + (3L - 5d)\times\left(\frac{L}{6}\right)\left(3 - \frac{5d}{L}\right) + 2d(L - d)\right]$$

$$= \frac{PL}{12}\left(3 - \frac{d}{L}\right)^2 = 78.4 \text{ kN} \cdot \text{m} \quad \leftarrow$$



Note: $R_A = \dfrac{P}{2}\left(3 + \dfrac{d}{L}\right) = 16$ kN

$R_B = \dfrac{P}{2}\left(3 - \dfrac{d}{L}\right) = 14$ kN